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Modified KDP Model on the Kagomé Lattice^{*)}

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The modified potassium dihydrogen phosphate (KDP) model on a Kagomé lattice was first considered by Miyazima and Syozi (MS).¹⁾ As in the case of a square lattice,^{2),3)} the partition function is evaluated by converting the model into a dimer problem. Miyazima and Syozi used the method of Ref. 2) and obtained the partition function after evaluating an 18×18 determinant.

Here we show that the procedure can be much simplified. In fact, the partition function of the modified KDP model on a Kagomé lattice can be readily derived in terms of established results. As a result, we obtain the partition function which is different from that of Miyazima and Syozi.^{**)} Our expression of the partition

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^{**)} We have since been informed by Dr. Syozi that an error was found in their original calculation. Their result after the correction now agrees with our expression (3).

function also permits a direct identification of the critical exponents α and α' .

We follow the notations of MS in defining the model. To evaluate the partition function, we use, however, the more direct dimer equivalence given in Ref. 3).^{*)} Following the same reasoning as discussed in Ref. 3), we may replace in the resulting terminal lattice each chain of three lattice edges with respective activities $z_1, 1, z_2$ by a single edge of activity $z_1 z_2$. We are then led to the dimer lattice L^d shown in Fig. 1. The partition function Z of the modified KDP model on the underlying Kagomé lattice (the dotted lines) is then equal to the dimer generating function of

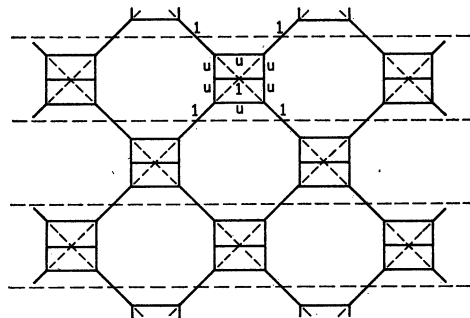


Fig. 1. The Kagomé lattice (dotted lines) and the equivalent dimer lattice L^d (heavy lines). The oblique dotted lines form a square lattice L_s .

^{*)} With the simplification of $u_2=1, u_3=u_4=u_5=u_6=u=e^{-\beta\epsilon}$.

L^4 with activities shown in the Figure.

Next we observe that the dimer problem for L^4 is precisely an eight-vertex problem⁴⁾ for the square lattice L_s defined by the oblique dotted lines. To be more precise, in the notations of Ref. 4), the eight-vertex model has the following vertex weights :*)

$$\begin{aligned}\omega_1 &= u^2 + 2u^3, \\ \omega_2 &= 1, \\ \omega_3 &= \omega_4 = u^2, \\ \omega_5 &= \omega_6 = u + u^2, \\ \omega_7 &= \omega_8 = 0.\end{aligned}\quad (1)$$

It is readily verified that these vertex weights satisfy the free fermion condition⁴⁾

$$\omega_1\omega_2 + \omega_3\omega_4 = \omega_5\omega_6 + \omega_7\omega_8. \quad (2)$$

The partition function is then given by Eq. (16) of Ref. 4). Using the vertex weights (1), we find

$$\lim_{n \rightarrow \infty} \frac{3}{n} \ln Z = \frac{1}{8\pi^2} \int_0^{2\pi} d\theta \int_0^{2\pi} d\varphi \ln D(\theta, \varphi) \quad (3)$$

where n is the number of vertices in the Kagomé lattice,**) and

$$\begin{aligned}D(\theta, \varphi) &= 4u^6 + 4u^5 [1 + \cos \theta + \cos \varphi] \\ &+ u^4 [3 + 2\cos \theta + 2\cos \varphi + 2\cos(\theta - \varphi)] \\ &- 4u^3 \cos(\theta + \varphi) \\ &- 2u^2 [\cos \theta + \cos \varphi + \cos(\theta + \varphi)] + 1.\end{aligned}$$

The transition occurs at $D(0, 0) = 0$ which leads to the critical condition $u = 1/2$, or

*) To obtain ω_1 , for example, we observe that when no dimers occupy the four edges leading into the dimer city at a vertex of L_s , there exist three allowed dimer configurations of weights u^2 , u^3 and u^4 , respectively within the city.

***) The number N in Ref. 1) is related to n by $n = 3N$.

$T_c = \varepsilon/k \ln 2$. This result agrees with that of Miyazima and Syozi. However, our expression of the partition function is different from that of MS. In the low temperature (small u) expansion, for example, the leading term in (3) is $2u^6$ while that of MS is $2u^5$.*)

The thermodynamic properties can be discussed on the basis of (3). The details are rather involved***) but, fortunately, the hard work has been done by Hurst and Green.⁵⁾ It turns out that the first derivative of (3), i.e., the energy, can be expressed in terms of the complete elliptic integrals of the first and third kinds. Consequently the specific heat diverges logarithmically both above and below T_c , and has the exponents $\alpha = \alpha' = 0$. It can be shown that the polarizability, when a field is included, has a similar divergence. It is of interest to note that, unlike in the case of a square lattice, the model is *not* frozen below T_c . This suggests that something interesting might happen in the Slater KDP (or the F) model on a Kagomé lattice. The latter problems, to our knowledge, have not been solved.

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- 1) S. Miyazima and I. Syozi, Prog. Theor. Phys. **40** (1968), 425.
- 2) F. Y. Wu, Phys. Rev. Letters **18** (1967), 605.
- 3) F. Y. Wu, Phys. Rev. **168** (1968), 539.
- 4) C. Fan and F. Y. Wu, Phys. Rev. **B2** (1970), 723.
- 5) H. S. Green and C. A. Hurst, *Order-Disorder Phenomena* (Interscience, New York, 1964). See, in particular, Sec. 8.4 (d).

*) See the second footnote on p. 2156.

***) Discussion in the footnote in the right-hand column on p. 2156 is simpler because of the further restriction $\omega_1\omega_2 = \omega_5\omega_6$ or $\omega_1\omega_2 = \omega_7\omega_8$.