

Experiment 19
Elastic Deformation

Your Name

Lab Partner: Lab Partner's Name

TA: TA's Name

Introduction

In this experiment, we will explore how stress and strain affect the sag of an aluminum "I" beam. In the first investigation, we will measure the dimensions of the "I" beam and predict the sag given the load and distance between the beam supports. In the second investigation, we measure the sag of the "I" beam under several different loads. Next, we plot the sag of the beam versus the load. Using the slope of the plot, dimensions of the beam, and the location of the supports, we'll calculate Young's modulus for aluminum. Using the accepted value for Young's modulus for aluminum, we can verify our result. If the accepted value for Young's modulus for aluminum falls within the uncertainty of our result, we will have experimentally verified the relationship between stress, strain, and Hooke's Law.

Investigation 1

Data and Analysis

d (mm)	δd (mm)	l (mm)	δl (mm)	t (mm)	δt (mm)	A' (mm ²)	$\delta A'$ (mm ²)
1.10	0.01	6.58	0.01	11.98	0.01	7.24	0.067
Y (N/m ²)	δY (N/m ²)	a (mm)	δa (mm)	L (m)	W (N)	h (μ m)	δh (μ m)
6.50E+10	5.00E+09	10.88	0.014	0.30	3.0	61	4.7

d (mm)	δd (mm)	l (mm)	δl (mm)	t (mm)	δt (mm)	A' (mm ²)	$\delta A'$ (mm ²)
1.10	0.01	6.58	0.01	11.98	0.01	7.24	0.067
Y (N/m ²)	δY (N/m ²)	a (mm)	δa (mm)	L (m)	W (N)	h (μ m)	δh (μ m)
6.50E+10	5.00E+09	10.88	0.014	0.40	3.0	144	11.1

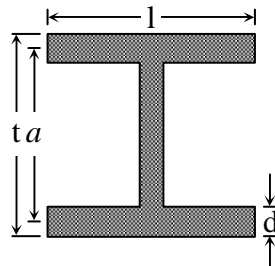


Figure of the "I" beam cross section.

δd , δl , δt was taken to be half the smallest division of the caliper. δY was found using the values from table 19.1 and the following equation:

$$\delta Y = \frac{Y_{\max} - Y_{\min}}{2}$$

A' and $\delta A'$ were found using the following equations:

$$A' = l * d$$

$$\delta A' = A' * \left(\left(\frac{\delta l}{l} \right)^2 + \left(\frac{\delta d}{d} \right)^2 \right)^{1/2} \text{ from Appendix A, page 5.}$$

a and δa were found using the following equations:

$$a = t - d$$

$$\delta a = \left((\delta t)^2 + (\delta d)^2 \right)^{1/2} \text{ from Appendix A, page 5.}$$

h and δh were found using the following equations:

$$h = \frac{WL^3}{24Ya^2A'}$$

$$\delta h = h * \left(\left(\frac{\delta Y}{Y} \right)^2 + \left(2 \frac{\delta a}{a} \right)^2 + \left(\frac{\delta A'}{A'} \right)^2 \right)^{1/2}$$

Our TA helped us with the δh formula. She showed us how it was just an extension of the formulas in Appendix A on page 5. Since L and W were numbers given to us without a stated error, I will assume L and W have negligible error and don't contribute any error to δh .

Investigation 2

Data, Graph, and Analysis

h (μm)	m (g)	W (N)
0	0	0
39	199.48	1.9549
78	399.48	3.9149
120	599.92	5.8792
158	799.20	7.8322

$\delta h = 5 \mu\text{m}$ (half the smallest division on the feeler guage).

$\delta m = 0.05 \text{ g}$ (half the smallest division on the scale).

$\delta L = 0.05 \text{ cm}$ (half the smallest division on the ruler).

$\delta g = 0 \text{ m/s}^2$ (assume the acceleration due to gravity has no error).

δW is so small it can be neglected.

See attached graph.

The slope of the best fit line is 20.3 $\mu\text{m}/\text{N}$. We determined the error in slope by drawing two other lines (largest and smallest) that would still fit the data within the error bars. Points A and B were used to find the slope of the largest line (see graph), and points C and D were used to find the slope of the smallest line (see graph). The coordinates of the points are below.

	W (N)	h (μm)
A	7.5	156
B	0.8	12
C	8.4	164
D	0.8	20

$$\text{Slope}_{AB} = 21.5 \mu\text{m}/\text{N}, \text{Slope}_{CD} = 18.9 \mu\text{m}/\text{N}$$

$$\delta\text{Slope} = (21.5 - 18.9)/2 = 1.3 \mu\text{m}/\text{N}$$

From the plot, find h with a load of 3.0 N and compare it to the h from investigation 1.

$$\text{Since } h = 20.267 * W - 0.3706, h = 20.267 * 3.0 - 0.3706 = 60.4 \mu\text{m}$$

Comparing that to the value in investigation 1:

$$\% \text{ difference} = \frac{|61 - 60.4|}{60.4} * 100 = 1.0\%$$

$$Y = \frac{L^3}{24(\text{slope})a^2 A'} = 6.48 * 10^{10} \text{ N}/\text{m}^2$$

$$\delta Y = Y * \left(\left(3 \frac{\delta L}{L} \right)^2 + \left(\frac{\delta \text{Slope}}{\text{Slope}} \right)^2 + \left(2 \frac{\delta a}{a} \right)^2 + \left(\frac{\delta A'}{A'} \right)^2 \right) = 2.63 * 10^8 \text{ N}/\text{m}^2$$

Our TA helped us with the δY formula. Again, it was just an extension of the formulas in Appendix A on page 5. The error in slope contributes the most error in δY . If I compare my measured Y to the average value for Y given in table 19.1, I get the following:

$$\% \text{ difference} = \frac{|6.5 * 10^{10} - 6.48 * 10^{10}|}{6.5 * 10^{10}} * 100 = 0.3\%$$

Conclusion

We experimentally verified the relationship between stress and strain for an I-beam. We learned why the shape of I-beams makes them strong and resistant to bending when the "I" is vertical. First we were able to theoretically calculate the sag of the aluminum I-beam based on the beam's dimensions. Our theoretical sag due to a given load was $61 \mu\text{m} \pm 4.7 \mu\text{m}$. The measured sag from our plot, under the same load, was $60.4 \mu\text{m}$. The measured sag is well within the uncertainty of the theoretical sag. Next, we found Young's modulus for aluminum. Our measured value was $6.48 \times 10^{10} \text{ N/m}^2 \pm 2.63 \times 10^8 \text{ N/m}^2$. This fell, within our measured experimental uncertainty, in the accepted range for Young's modulus given to us in table 19.1, $6 \times 10^{10} \text{ N/m}^2$ to $7 \times 10^{10} \text{ N/m}^2$.

Questions

- $$s = \frac{T}{A} = \frac{(10\text{kg})(9.8\text{m/s}^2)}{\pi(0.1\text{mm})^2} = 3119 \text{ N/mm}^2 = 3.12 \times 10^9 \text{ N/m}^2$$
 - $$\Delta L = \frac{SL}{Y} = \frac{(3.12 \times 10^9 \text{ N/m}^2)(5\text{m})}{1.2 \times 10^{11} \text{ N/m}^2} = 1.30 \times 10^{-13} \text{ m}$$
 - $$\% \text{ difference} = \frac{|3.12 \times 10^9 \text{ N/m}^2 - 5 \times 10^8 \text{ N/m}^2|}{5 \times 10^8 \text{ N/m}^2} * 100 = 524\%$$
- $$S_{av} = \frac{WL}{8aA'} = \frac{(7.83 \text{ N})(0.30 \text{ m})}{8(0.01088 \text{ m})(7.24 \times 10^{-6} \text{ m}^2)} = 3.73 \times 10^6 \text{ N/m}^2$$
$$\% \text{ difference} = \frac{|2 \times 10^8 \text{ N/m}^2 - 3.76 \times 10^6 \text{ N/m}^2|}{2 \times 10^8 \text{ N/m}^2} * 100 = 98.1\%$$

$$h = 20.267W - 0.3706$$

Sag of Aluminum "I" Beam

